

On graphs admitting two disjoint maximum independent sets*

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Abstract

An independent set S is *maximal* if it is not a proper subset of an independent set, while S is *maximum* if it has a maximum size. The problem of whether a graph has a pair of disjoint maximal independent sets was introduced by Berge in the early 1970s. The class of graphs for which every induced subgraph admits two disjoint maximal independent sets was characterized by Schaudt in 2015. In this paper, we are focused on finding conditions ensuring the existence of two disjoint maximum independent sets.

Keywords: Maximum independent set, shedding vertex, König-Egervàry graph, almost bipartite graph, unicyclic graph.

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Grafi z dvema disjunktnima največjima neodvisnima množicama*

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Povzetek

Neodvisna množica S je *maksimalna*, če ni prava podmnožica neodvisne množice, medtem ko je S *največja* neodvisna množica, če ima največjo velikost. Problem, kdaj ima graf dve disjunktni maksimalni neodvisni množici, je zastavil Berge v zgodnjih 1970-ih. Razred grafov, za katere velja, da ima vsak induciran podgraf dve disjunktni maksimalni neodvisni množici, je karakteriziral Schaudt leta 2015. V tem članku se osredotočamo na iskanje pogojev, ki zagotavljajo obstoj dveh disjunktnih največjih neodvisnih množic.

Ključne besede: Največja neodvisna množica, osipna točka, König-Egervàryjev graf, skoraj dvodelni graf, enociklični graf.

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