

Chromatic uniqueness of zero-divisor graphs*

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Abstract

The *zero-divisor graph* $\Pi(R)$ of a commutative ring R is the graph whose vertices are the elements of R such that the vertices u and v are adjacent if and only if $uv = 0$. If the graphs G and H have the same chromatic polynomial, then we say that they are *chromatically equivalent* (or χ -*equivalent*), written as $G \sim H$. Suppose a graph is uniquely determined by its chromatic polynomial. Then it is said to be *chromatically unique* (or χ -*unique*).

In this paper, we discuss the question: For which numbers n is the graph $\Pi(Z_n)$ χ -unique?

While Z_n is one of the simplest rings, we proved that for any graph A_0 , for some n , $\Pi(Z_n)$ contains an induced subgraph isomorphic to A_0 . The first result in the subject states that for $n \geq 10$ even, $\Pi(Z_n)$ is not χ -unique (Gehet, Khalaf). By definition, n is *square-free* if it is prime or the product of different prime numbers. Our main result is the following. If $n \geq 10$ is neither square-free nor the square of a prime then it is not χ -unique. Here and in our preceding work, we use a common method.

For odd square-free non-prime n , the problem is open, though on the structure of $\Pi(Z_n)$ we know much in this case.

Keywords: Zero-divisor graph, chromatic equivalence, chromatic uniqueness.

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Kromatska unikatnost grafov deliteljev ničā*

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Povzetek

Graf deliteljev ničā $\Pi(R)$ komutativnega kolobarja R je graf, katerega vozlišča so elementi kolobarja R , pri čemer sta vozlišči u in v sosedni, če in samo če je $uv = 0$. Če imata grafa G in H isti kromatski polinom, pravimo, da sta *kromatsko ekvivalentna* (ali χ -ekvivalentna), kar zapišemo $G \sim H$. Denimo, da je graf enolično določen s svojim kromatskim polinomom. Potem rečemo, da je *kromatsko unikatnen* (ali χ -unikaten).

V tem članku obravnavamo vprašanje: za katera števila n je graf $\Pi(Z_n)$ χ -unikaten?

Čeprav je Z_n eden izmed najenostavnejših kolobarjev, smo dokazali, da za poljuben graf A_0 , za neki n , $\Pi(Z_n)$ vsebuje nek induciran podgraf, izomorfen grafu A_0 . Prvi rezultat v zvezi s tem pove: če je $n \geq 10$ sodo število, $\Pi(Z_n)$ ni χ -unikaten (Gehet, Khalaf). Po definiciji je število n brez kvadratov, če je bodisi praštevilo bodisi produkt različnih praštevil. Naš glavni rezultat je naslednji. Če $n \geq 10$ ni niti brez kvadrata niti ni praštevilo, potem $\Pi(Z_n)$ ni χ -unikaten. V tem članku in v našem prejšnjem članku uporabljamo skupno metodo.

Za liho število n , ki ni brez kvadratov in ni praštevilo, ostaja problem odprt, pa čeprav v tem primeru o strukturi $\Pi(Z_n)$ vemo veliko.

Ključne besede: Graf deliteljev ničā, kromatska ekvivalenca, kromatska enoličnost.

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