

Permutation graphs and the weak Bruhat order*

Richard A. Brualdi 

Department of Mathematics, University of Wisconsin, Madison, WI 53706, USA

Geir Dahl[†] 

Department of Mathematics, University of Oslo, Norway

Received 23 March 2022, accepted 5 January 2023, published online 27 February 2023

Abstract

Permutation graphs are graphs associated with permutations where edges represent inversions. We study different classes of permutation graphs and isomorphic permutation graphs. A complete answer of a basic isomorphism question is given for trees. A connection to the majorization order for integer partitions is established, in connection with the notion of inversion vectors. Finally, we study special transitivity properties related to inversions of permutations.

Keywords: Permutation, inversion, partially ordered set (poset), comparability graph, permutation graph, weak Bruhat order.

Math. Subj. Class.: 05A05, 05B20, 05C05, 05C38

*The authors are grateful to the referee for several useful comments.

[†]Corresponding author.

E-mail addresses: brualdi@math.wisc.edu (Richard A. Brualdi), geird@math.uio.no (Geir Dahl)



Permutacijski grafi in šibki Bruhatov red*

Richard A. Brualdi 

Department of Mathematics, University of Wisconsin, Madison, WI 53706, USA

Geir Dahl† 

Department of Mathematics, University of Oslo, Norway

Prejeto 23. marca 2022, sprejeto 5. februarja 2023, objavljeno na spletu 27. februarja 2023

Povzetek

Permutacijski grafi so grafi, povezani s permutacijami, pri čemer povezave predstavljajo inverzije. V članku raziskujemo različne razrede permutacijskih grafov ter izomorfnih permutacijskih grafov. Podamo popoln odgovor na osnovno vprašanje izomorfizma za razred dreves. Vzpostavimo povezavo z majorizacijskim redom za celoštevilске particije v povezavi z inverzijskimi vektorji. Raziščemo tudi posebne tranzitivnostne lastnosti, povezane z inverzijami permutacij.

Ključne besede: Permutacija, inverzija, delno urejena množica, primerljivostni graf, permutacijski graf, šibki Bruhatov red.

Math. Subj. Class.: 05A05, 05B20, 05C05, 05C38

*Avtorja sta hvaležna recenzentu za številne koristne pripombe.

†Kontaktni avtor.

E-poštna naslova: brualdi@math.wisc.edu (Richard A. Brualdi), geird@math.uio.no (Geir Dahl)