

# Semi-perimeter and inner site-perimeter of $k$ -ary words and bargraphs

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## Abstract

Given a bargraph  $B$ , a *border cell* of  $B$  is a cell of  $B$  that shares at least one common edge with an outside cell of  $B$ . Clearly, the inner site-perimeter of  $B$  is the number of border cells of  $B$ . A *tangent cell* of  $B$  is a cell of  $B$  which is not a border cell of  $B$  and shares at least one vertex with an outside cell of  $B$ . In this paper, we study the generating function for the number of  $k$ -ary words, represented as bargraphs, according to the number of horizontal steps, up steps, border cells and tangent cells. This allows us to express some cases via Chebyshev polynomials of the second kind. Moreover, we find an explicit formula for the number of bargraphs according to the number of horizontal steps, up steps, and tangent cells/inner site-perimeter. We also derive asymptotic estimates for the mean number of tangent cells/inner site-perimeter.

*Keywords:* Bargraphs, Chebyshev polynomials,  $k$ -ary words, semi-perimeter, inner site-perimeter.

*Math. Subj. Class.:* 05A15, 05A16, 60C05

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# Polovični obseg in notranji obodni obseg besed iz nabora $k$ znakov in prečnih grafov

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## Povzetek

Če je dan prečni graf  $B$ , je *mejna celica* grafa  $B$  takšna njegova celica, ki ima najmanj eno skupno povezavo z neko njegovo zunanjo celico. Notranji obodni obseg grafa  $B$  je število mejnih celic grafa  $B$ . *Tangentna celica* grafa  $B$  je takšna njegova celica, ki ni njegova mejna celica in si deli najmanj eno vozlišče z neko njegovo zunanjo celico. V tem članku preučujemo rodovno funkcijo za število besed iz nabora  $k$  znakov, predstavljenih s prečnimi grafi, glede na število vodoravnih korakov, korakov navzgor, mejnih celic in tangentnih celic. To nam omogoča izraziti rešitve v nekaterih primerih s pomočjo polinomov Čebiševa druge vrste. Poleg tega najdemo eksplicitno formulo za število prečnih grafov glede na število vodoravnih korakov, korakov navzgor in tangentnih celic oz. notranjega obodnega obsega. Izpeljemo tudi asimptotske ocene za povprečno število tangentnih celic oz. za notranji obodni obseg.

*Ključne besede:* Prečni grafi, polinomi Čebiševa, besede iz nabora  $k$  znakov, polovični obseg, notranji obodni obseg.

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