

C_4 -face-magic toroidal labelings on $C_m \times C_n$

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Abstract


For a graph $G = (V, E)$ naturally embedded in the torus, let $\mathcal{F}(G)$ denote the set of faces of G . Then, G is called a C_n -face-magic toroidal graph if there exists a bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ such that for every $F \in \mathcal{F}(G)$ with $F \cong C_n$, the sum of all the vertex labels along C_n is a constant S . Let $x_v = f(v)$ for all $v \in V(G)$. We call $\{x_v : v \in V(G)\}$ a C_n -face-magic toroidal labeling on G . We show that, for all $m, n \geq 2$, $C_m \times C_n$ admits a C_4 -face-magic toroidal labeling if and only if either $m = 2$, or $n = 2$, or both m and n are even. We say that a C_4 -face-magic toroidal labeling $\{x_{i,j} : (i, j) \in V(C_{2m} \times C_{2n})\}$ on $C_{2m} \times C_{2n}$ is antipodal balanced if $x_{i,j} + x_{i+m,j+n} = \frac{1}{2}S$, for all $(i, j) \in V(C_{2m} \times C_{2n})$. We show that there exists an antipodal balanced C_4 -face-magic toroidal labeling on $C_{2m} \times C_{2n}$ if and only if the parity of m and n are the same. Furthermore, when both m and n are even, an antipodal balanced C_4 -face-magic toroidal labeling on $C_{2m} \times C_{2n}$ is both row-sum balanced and column-sum balanced. In addition, when $m = n$ is even, an antipodal balanced C_4 -face-magic toroidal labeling on $C_{2n} \times C_{2n}$ is diagonal-sum balanced.

Keywords: C_4 -face-magic graphs, polyomino, toroidal graphs, Cartesian products of cycles.

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C_4 -lična magična torusna številčenja grafa $C_m \times C_n$

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Povzetek

Za graf $G = (V, E)$, naravno vložen na torus, naj $\mathcal{F}(G)$ označuje množico lic, določenih s to vložitvijo grafa G . Tako vloženi graf G se imenuje C_n -lični magični torusni graf, če obstaja takšna bijekcija $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$, da je za vsako lice $F \in \mathcal{F}(G)$, ki zadošča pogoju $F \cong C_n$, vsota vseh oznak točk vzdolž C_n enaka konstanti S . Naj bo $x_v = f(v)$ za vse $v \in V(G)$. Množico $\{x_v : v \in V(G)\}$ imenujemo C_n -lično magično torusno številčenje grafa G . Dokažemo, da za vse celoštevilске vrednosti $m, n \geq 2$, graf $C_m \times C_n$ dopušča C_4 -lično magično torusno številčenje natanko takrat, ko je bodisi $m = 2$ bodisi $n = 2$, ali pa sta tako m kot n sodi števili. Pravimo, da je C_4 -lično magično torusno številčenje $\{x_{i,j} : (i,j) \in V(C_{2m} \times C_{2n})\}$ grafa $C_{2m} \times C_{2n}$ antipodno uravnoreženo, če je $x_{i,j} + x_{i+m,j+n} = \frac{1}{2}S$ za vse točke $(i,j) \in V(C_{2m} \times C_{2n})$. Dokažemo, da obstaja antipodno uravnoreženo C_4 -lično magično torusno številčenje grafa $C_{2m} \times C_{2n}$ natanko takrat, ko sta števili m in n obe sodi ali obe lihi. Nadalje, kadar sta tako m kot n soda, je antipodno uravnoreženo C_4 -lično magično torusno številčenje grafa $C_{2m} \times C_{2n}$ hkrati uravnoreženo tako v smislu vrstičnih vsot kot tudi vsot stolpcev. Poleg tega, če je $m = n$ sod, je antipodno uravnoreženo C_4 -lično magično torusno številčenje grafa $C_{2n} \times C_{2n}$ uravnoreženo tudi v smislu vsot po diagonalah.

Ključne besede: C_4 -lični magični grafi, poliomino, torusni grafi, kartezični produkt ciklov.

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