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On polyhedral realizations of Hurwitz's regular map $\{3,7\}_{18}$ of genus 7 with geometric symmetries*

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Abstract

In 2017 a first selfintersection-free polyhedral realization of Hurwitz's regular map $\{3,7\}_{18}$ of genus 7 was found by Michael Cuntz and the first author. For any regular map which had previously been realized as a polyhedron without self-intersections in 3-space, it was also possible to find such a polyhedron with nontrivial geometric symmetries. So it is natural to ask of whether we can find for the above-mentioned regular map a corresponding version with some non-trivial geometric symmetry. The orientation-preserving combinatorial automorphism group of this Hurwitz map is the projective special linear group $\mathbf{PSL}(2,8)$ of order $504 = 2^3 \cdot 3^2 \cdot 7$. All non-trivial subgroups of $\mathbf{PSL}(2,8)$ are candidates for such a geometric symmetry. Using the GAP software for exploring the subgroup structure, we found that it is sufficient to consider only four cyclic subgroups whose order is 9, 7, 3, and 2, respectively. We prove that there are obstructions for selfintersection-free polyhedral realizations of the Hurwitz map $\{3,7\}_{18}$ of genus 7 with geometric rotational symmetries of order 9 or 3. We provide new small integer coordinates within the realization space known from 2017, which are also suitable for making a 3D-printed model. We present Kepler–Poinsot type realizations, both with 7-fold and with 3-fold rotational symmetry, the latter with integer coordinates.

IN MEMORY OF BRANKO GRÜNBAUM.

Keywords: Hurwitz surface, regular map, Kepler–Poinsot type polyhedron.

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O poliedrskih realizacijah Hurwitzevega pravilnega zemljevida $\{3,7\}_{18}$ rodu 7, ki imajo geometrijske simetrije*

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Povzetek

Leta 2017 sta Michael Cuntz in prvi avtor članka našla prvo poliedrsko realizacijo Hurwitzevega pravilnega zemljevida $\{3,7\}_{18}$ rodu 7, ki nima nobenih samopresečišč. Za vsak pravilni zemljevid, ki je bil doslej realiziran kot polieder brez samopresečišč v tridimenzionalnem prostoru, je bilo mogoče najti tudi takšno poliedrsko realizacijo, ki ima netrivialne geometrijske simetrije. Tako se naravno zastavlja vprašanje, ali lahko najdemo za zgoraj omenjeni pravilni zemljevid ustrezno verzijo z neko netrivialno geometrijsko simetrijo. Grupa tistih kombinatoričnih avtomorfizmov tega pravilnega ˇ Hurwiczevega zemljevida, ki ohranjajo orientacijo, je projektivna posebna linearna grupa $\mathbf{PSL}(2,8)$ reda $504 = 2^3 \cdot 3^2 \cdot 7$. Vse netrivialne podgrupe grupe $\mathbf{PSL}(2,8)$ bi načeloma lahko imele takšno geometrijsko simetrijo. S pomočjo programskega orodja GAP, ki omogoča raziskovanje strukture podgrup, pa sva ugotovila, da zadostuje, če obravnavamo samo štiri ciklične podgrupe, in sicer tiste, katerih red je 9, 7, 3 in 2. Predstaviva ovire, ki preprečujejo obstoj takšnih poliedrskih realizacij Hurwitzevega zemljevida $\{3,7\}_{18}$ rodu 7 brez samopresečišč, ki bi imele geometrijske rotacijske simetrije reda 9 ali 3. Predstavimo nove majhne celoštevilske koordinate znotraj realizacijskega prostora, obravnavanega v zgoraj omenjenem članku iz leta 2017, ki so primerne tudi za izdelavo ustreznega modela s 3D-tiskalnikom. Predstaviva tudi realizaciji Kepler–Poinsotovega tipa, ki imata 7-kratno oz. 3-kratno rotacijsko simetrijo; za to slednjo določiva tudi njene celoštevilske koordinate.

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V SPOMIN NA BRANKA GRÜNBAUMA.

Ključne besede: Hurwitzeva ploskev, pravilni zemljevud, polieder Kepler–Poinsotovega tipa.
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