

# Realizations of lattice quotients of Petrie-Coxeter polyhedra\*

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## Abstract

The Petrie-Coxeter polyhedra naturally give rise to several infinite families of finite regular maps on closed surfaces embedded into the 3-torus. For the dual pair of Petrie-Coxeter polyhedra  $\{4, 6 | 4\}$  and  $\{6, 4 | 4\}$ , we describe highly-symmetric embeddings of these maps as geometric, combinatorially regular polyhedra (polyhedral 2-manifolds) with convex faces in Euclidean spaces of dimensions 5 and 6. In each case the geometric symmetry group is a subgroup of index 1 or 2 in the combinatorial automorphism group.

IN MEMORY OF BRANKO GRÜNBAUM.

*Keywords: Regular polyhedron, regular map, Petrie-Coxeter polyhedron, polyhedral embedding, polyhedral 2-manifold, automorphism group.*

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# Prikazi mrežnih kvocientov Petrie-Coxeterjevih poliedrov\*

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## Povzetek

Petrie-Coxeterjevi poliedri naravno porodijo številne neskončne družine končnih pravilnih zemljevidov na sklenjenih ploskvah, vloženi v 3-torus. Za dualni par Petrie-Coxeterjevih poliedrov  $\{4, 6 \mid 4\}$  in  $\{6, 4 \mid 4\}$  opišemo visoko simetrične vložitve teh zemljevidov kot geometrijske, kombinatorično pravilne poliedre (poliedrske 2-mnogoterosti) s konveksnimi lici v evklidskih prostorih dimenzij 5 in 6. V obeh primerih je geometrijska simetrijska grupa podgrupa z indeksom 1 ali 2 v kombinatorični grupi avtomorfizmov.

## V SPOMIN NA BRANKA GRÜNBAUMA.

*Ključne besede: Pravilni polieder, pravilen zemljevid, Petrie-Coxeterjev polieder, poliedrska vložitev, poliedrka 2-mnogoterost, grupa avtomorfizmov.*

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