


Flat extensions of abstract polytopes

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Abstract


We consider the problem of constructing an abstract $(n+1)$ -polytope \mathcal{Q} with k facets isomorphic to a given n -polytope \mathcal{P} , where $k \geq 3$. In particular, we consider the case where we want \mathcal{Q} to be $(n-2, n)$ -flat, meaning that every $(n-2)$ -face is incident to every n -face (facet). We show that if \mathcal{P} admits such a *flat extension* for a given k , then the facet graph of \mathcal{P} is $(k-1)$ -colorable. Conversely, we show that if the facet graph is $(k-1)$ -colorable and $k-1$ is prime, then \mathcal{P} admits a flat extension for that k . We also show that if \mathcal{P} is facet-bipartite, then for every even k , there is a flat extension $\mathcal{P}|k$ such that every automorphism of \mathcal{P} extends to an automorphism of $\mathcal{P}|k$. Finally, if \mathcal{P} is a facet-bipartite n -polytope and \mathcal{Q} is a vertex-bipartite m -polytope, we describe a *flat amalgamation* of \mathcal{P} and \mathcal{Q} , an $(m+n-1)$ -polytope that is $(n-2, n)$ -flat, with n -faces isomorphic to \mathcal{P} and co- $(n-2)$ -faces isomorphic to \mathcal{Q} .

Keywords: Polytope, extension, amalgamation, perfect 1-factorization.

Math. Subj. Class.: 52B05, 52B11, 52B15



Ploskovite razširitve abstraktnih politopov

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Povzetek

Obravnavamo problem konstruiranja abstraktnega $(n+1)$ -politopa \mathcal{Q} s k fasetami, izomorfni danemu n -politopu \mathcal{P} , pri čemer je $k \geq 3$. Še posebej nas zanima primer, ko je politop \mathcal{Q} $(n-2, n)$ -ploskovit, kar pomeni, da je vsako $(n-2)$ -lice incidentno vsakemu n -licu (faseti). Pokažemo, da če ima \mathcal{P} takšno *ploskovito razširitev* za dani k , potem se da fasetni graf politopa \mathcal{P} pobarvati s $(k-1)$ barvami. In obratno: če je fasetni graf $(k-1)$ -barven in je $k-1$ praštevilo, potem ima \mathcal{P} ploskovito razširitev pri tem k . Pokažemo tudi: če je \mathcal{P} fasetno dvodelen, potem za vsako sodo število k obstaja takšna ploskovita razširitev $\mathcal{P}|k$, da se da vsak avtomorfizem politopa \mathcal{P} razširiti do avtomorfizma njegove razširitve $\mathcal{P}|k$. Nazadnje obravnavamo primer, ko je \mathcal{P} fasetno dvodelen n -politop, \mathcal{Q} pa točkovno dvodelen m -politop; tedaj lahko opišemo *ploskovito združitev* politopov \mathcal{P} in \mathcal{Q} , to je $(m+n-1)$ -politop, ki je $(n-2, n)$ -ploskovit in ima n lic izomorfni \mathcal{P} in n lic izomorfni \mathcal{Q} .

Ključne besede: Politop, razširitev, združitev, popolna 1-faktorizacija.

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