


The $T(5)$ property of packings of squares*

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Abstract

According to a classical theorem of Gruenbaum, if any five of a family of pairwise disjoint translates of a square has a transversal line (the family satisfies $T(5)$), then the whole family has a transversal line (satisfies T). First we show that this result is optimal in the sense that the “ $T(5)$ implies T ” property does not necessarily hold anymore if only the slightly shrunk versions of the squares are pairwise disjoint. Next we prove the “ $T(5)$ implies T ” property for a family of translates of squares if the interiors are pairwise disjoint and there exist two translates meeting at a common vertex.

Keywords: Transversals, parallelograms, Minkowski plane.

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$T(5)$ lastnost razporeditev kvadratov*

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Povzetek

Po klasičnem izreku Gruenbauma velja: če za katerihkoli pet elementov iz družine paroma tujih kvadratov, ki jih dobimo z vzporednimi premiki danega kvadrata, obstaja njihova prečnica (v tem primeru pravimo, da ima ta družina lastnost $T(5)$), potem ima celotna družina prečnico (zadošča lastnosti T). Najprej pokažemo, da je ta rezultat optimalen v tem smislu, da lastnost " $T(5)$ implicira T " ne drži več nujno, če so paroma tuje samo malce skršene različice kvadratov. Nadalje dokažemo, da lastnost " $T(5)$ implicira T " velja za družino translahiranih kopij danega kvadrata, katerih notranjosti so paroma tuje, in v kateri obstajata dva kvadrata, ki se sekata v skupnem oglišču.

Ključne besede: Prečnice, paralelogrami, ravnina Minkowskega.

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