

# An infinite family of incidence geometries whose incidence graphs are locally $X^*$

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## Abstract

We construct a new infinite family of incidence geometries of arbitrarily large rank. These geometries are thick and residually connected and their type-preserving automorphism groups are symmetric groups. We also compute their Buekenhout diagram. The incidence graphs of these geometries are locally  $X$  graphs, but more interestingly, the automorphism groups act transitively, not only on the vertices, but more strongly on the maximal cliques of these graphs.

DEDICATED TO THE MEMORY OF BRANKO GRÜNBAUM.

*Keywords: Kneser graph, locally  $X$  graph, incidence geometry.*

*Math. Subj. Class.: 05C75, 51E24*

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# Neskončna družina incidenčnih geometrij, katerih incidenčni grafi so lokalno $X^*$

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## Povzetek

Konstruiramo novo neskončno družino incidenčnih geometrij poljubno velikega ranga. Te geometrije so masivne in residualno povezane, njihove grupe tistih avtomorfizmov, ki ohranjajo tip, pa so simetrične. Izračunamo tudi njihov Buekenhoutov diagram. Incidenčni grafi teh geometrij so lokalno  $X$  grafi, zanimivo pa je tudi, da njihove grupe avtomorfizmov delujejo tranzitivno, in to ne samo na točkah, ampak širše – na maksimalnih klikah teh grafov.

POSVEČENO SPOMINU NA BRANKA GRÜNBAUMA.

*Ključne besede:* Kneserjev graf, lokalno  $X$  graf, incidenčna geometrija.

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