

# On the anti-Kekulé problem of cubic graphs\*

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## Abstract

An edge set  $S$  of a connected graph  $G$  is called an anti-Kekulé set if  $G - S$  is connected and has no perfect matchings, where  $G - S$  denotes the subgraph obtained by deleting all edges in  $S$  from  $G$ . The anti-Kekulé number of a graph  $G$ , denoted by  $ak(G)$ , is the cardinality of a smallest anti-Kekulé set of  $G$ . It is NP-complete to determine the anti-Kekulé number of a graph. In this paper, we show that the anti-Kekulé number of a 2-connected cubic graph is either 3 or 4, and the anti-Kekulé number of a connected cubic bipartite graph is always equal to 4. Furthermore, a polynomial time algorithm is given to find all smallest anti-Kekulé sets of a connected cubic graph.

*Keywords:* Anti-Kekulé set, anti-Kekulé number, cubic graphs.

*Math. Subj. Class.:* 05C10, 05C70, 05C90

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# O anti-Kekuléjevem problemu kubičnih grafov\*

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## Povzetek

Množica povezav  $S$  povezanega grafa  $G$  se imenuje anti-Kekuléjeva množica, če je  $G - S$  povezan graf brez popolnega prirejanja, kjer  $G - S$  označuje podgraf, dobljen z brisanjem vseh povezav množice  $S$  iz  $G$ . Anti-Kekuléjevo število grafa  $G$ , označeno z  $ak(G)$ , je moč najmanjše anti-Kekuléjeve množice grafa  $G$ . Problem določitve anti-Kekuléjevega števila grafa je NP-poln problem. V tem članku pokažemo, da je anti-Kekuléjevo število 2-povezanega kubičnega grafa bodisi 3 bodisi 4 in da je anti-Kekuléjevo število povezanega kubičnega dvodelnega grafa vedno enak 4. Predstavimo tudi algoritem, ki v polinomskem času najde vse najmanjše anti-Kekuléjeve množice povezanega kubičnega grafa.

*Ključne besede:* Anti-Kekuléjeva množica, anti-Kekuléjevo število, kubični grafi.

*Math. Subj. Class.:* 05C10, 05C70, 05C90

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