

# Axiomatic characterization of transit functions of weak hierarchies\*

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## Abstract

Transit functions provide a unified approach to study notions of intervals, convexities, and betweenness. Recently, their scope has been extended to certain set systems associated with clustering. We characterize here the class of set systems that correspond to  $k$ -ary monotonic transit functions. Convexities form a subclass and are characterized in terms of transit functions by two additional axioms. We then focus on axiom systems associated with weak hierarchies as well as other generalizations of hierarchical set systems.

*Keywords: Transit functions, convexities, weak hierarchies, axiom systems.*

*Math. Subj. Class.: 05C05, 05C99, 52A01*

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# Aksiomatična karakterizacija funkcij prehoda šibkih hierarhij\*

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## Povzetek

Funkcije prehoda omogočajo poenoten pristop k raziskovanju pojmov intervalov, konveksnosti in vmesnosti. Nedavno je bilo njihovo področje uporabe razširjeno na določene sisteme množic, ki so povezane z grozdenjem. Tukaj karakteriziramo razred sistemov množic, ki ustrezajo monotonim funkcijam prehoda s  $k$  argumenti. Konveksnosti tvorijo pod-razred tega razreda; karakteriziramo jih s pomočjo funkcij prehoda z dvema dodatnima aksiomoma. Nato se osredotočimo na aksiomatične sisteme, ki so povezani s šibkimi hierarhijami, pa tudi na druge posplošitve hierarhičnih sistemov množic.

*Ključne besede: Funkcije prehoda, konveksnosti, šibke hierarhije, aksiomatični sistemi.*

*Math. Subj. Class.: 05C05, 05C99, 52A01*

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