

Most rigid representations and Cayley index*

Joy Morris , Josh Tymburski

*Department of Mathematics and Computer Science, University of Lethbridge,
Lethbridge, AB T1K 3M4, Canada*

Received 27 March 2017, accepted 3 January 2018, published online 6 February 2018

Abstract

For any finite group G , a natural question to ask is the order of the smallest possible automorphism group for a Cayley graph on G . A particular Cayley graph whose automorphism group has this order is referred to as an MRR (Most Rigid Representation), and its Cayley index is a numerical indicator of this value. Study of GRRs showed that with the exception of two infinite families and thirteen individual groups, every group admits a Cayley graph whose MRR is a GRR, so that the Cayley index is 1. The full answer to the question of finding the smallest possible Cayley index for a Cayley graph on a fixed group was almost completed in previous work, but the precise answers for some finite groups and one infinite family of groups were left open. We fill in the remaining gaps to completely answer this question.

Keywords: Cayley graph, Cayley index, GRR, MRR, automorphisms.

Math. Subj. Class.: 05C25

*The authors would like to thank the anonymous referee whose careful reading and generous advice helped us to correct some errors and historical inaccuracies, and improved the exposition of this paper.

This research was supported in part by the Natural Science and Engineering Research Council of Canada, Grant RGPIN-2011-238552.

E-mail addresses: joy.morris@uleth.ca (Joy Morris), josh.tymburski@uleth.ca (Josh Tymburski)



Najrigidnejše reprezentacije in Cayleyjev indeks*

Joy Morris, Josh Tymburski

*Department of Mathematics and Computer Science, University of Lethbridge,
Lethbridge, AB T1K 3M4, Canada*

Prejeto 27. marca 2017, sprejeto 3. januarja 2018, objavljeno na spletu 6. februarja 2018

Povzetek

Za vsako končno grupo G se je naravno vprašati, kolikšen je red najmanjše možne grupe avtomorfizmov Cayleyjevega grafa na G . Poseben Cayleyjev graf, katerega grupa avtomorfizmov ima ta red, se imenuje MRR (najbolj rigidna reprezentacija), in njegov Cayleyjev indeks je numerečni indikator te vrednosti. Raziskave GRR-ov so pokazale, da – z izjemo dveh neskončnih družin in trinajstih posamičnih grup – vsaka grupa premore Cayleyjev graf, katerega MRR je GRR, tako da je njen Cayleyjev indeks 1. Odgovor na vprašanje določitve najmanjšega možnega Cayleyjevega indeksa za Cayleyjev graf na fiksni grupi je bil skoraj v celoti podan v prejšnjem delu, toda natančni odgovori za nekaj končnih grup in eno neskončno družino grup še niso bili najdeni. V tem članku zapolnimo preostale vrzeli in s tem popolnoma odgovorimo na to vprašanje.

Ključne besede: Cayleyjev graf, Cayleyjev indeks, GRR, MRR, avtomorfizem.

Math. Subj. Class.: 05C25

*Avtorja bi se rada zahvalila anonimnemu recenzentu, čigar skrbno branje in velikodušni nasveti so nama pomagali popraviti nekaj napak in zgodovinskih netočnosti ter izboljšati predstavitev tega prispevka.

Ta raziskava je bila delno podprta s strani Natural Science and Engineering Research Council of Canada, Grant RGPIN-2011-238552.

E-poštna naslova: joy.morris@uleth.ca (Joy Morris), josh.tymburski@uleth.ca (Josh Tymburski)