

# The $k$ -independence number of graph products\*

Yaping Mao

*School of Mathematics and Statistics, Qinghai Normal University,  
Xining, Qinghai 810008, China*

Eddie Cheng

*Department of Mathematics and Statistics, Oakland University, Rochester, MI USA 48309*

Zhao Wang<sup>†</sup>

*School of Mathematical Sciences, Beijing Normal University, Beijing 100875, China*

Zhiwei Guo

*School of Mathematics and Statistics, Qinghai Normal University,  
Xining, Qinghai 810008, China*

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## Abstract

The concept of  $k$ -independence number is a natural generalization of classical independence number. A  $k$ -independent set is a set of vertices whose induced subgraph has maximum degree at most  $k$ . The  $k$ -independence number of  $G$ , denoted by  $\alpha_k(G)$ , is defined as the maximum cardinality of a  $k$ -independent set of  $G$ . In this paper, we study the  $k$ -independence number on the lexicographical, strong, Cartesian and direct product and present several upper and lower bounds for these products of graphs.

*Keywords: Independence number,  $k$ -independent set,  $k$ -independence number, lexicographical product, strong product, Cartesian product, direct product.*

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<sup>†</sup>Corresponding author.

*E-mail addresses:* maoyaping@ymail.com (Yaping Mao), echeng@oakland.edu (Eddie Cheng), wangzhao@mail.bnu.edu.cn (Zhao Wang), guozhiweic@yahoo.com (Zhiwei Guo)

# $k$ -neodvisnost grafovskih produktov\*

Yaping Mao

*School of Mathematics and Statistics, Qinghai Normal University,  
Xining, Qinghai 810008, China*

Eddie Cheng

*Department of Mathematics and Statistics, Oakland University, Rochester, MI USA 48309*

Zhao Wang †

*School of Mathematical Sciences, Beijing Normal University, Beijing 100875, China*

Zhiwei Guo

*School of Mathematics and Statistics, Qinghai Normal University,  
Xining, Qinghai 810008, China*

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## Povzetek

Koncept  $k$ -neodvisnosti je naravna posplošitev klasične neodvisnosti.  $k$ -neodvisna množica vozlišč grafa  $G$  je taka podmnožica vozlišč grafa  $G$ , da ima podgraf grafa  $G$ , induciran na tej množici, maksimalno stopnjo največ  $k$ .  $k$ -neodvisnost grafa  $G$ , oznaka  $\alpha_k(G)$ , je definirana kot maksimalna moč  $k$ -neodvisne množice grafa  $G$ . V tem članku raziskujemo  $k$ -neodvisnost leksikografskega, krepkega, kartezičnega in direktnega produkta ter predstavimo več zgornjih in spodnjih mej za te grafovske produkte.

*Ključne besede:* Neodvisnost,  $k$ -neodvisna množica,  $k$ -neodvisnost, leksikografski produkt, krepki produkt, kartezični produkt, direktni produkt.

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†Kontaktni avtor.

*E-poštni naslovi:* maoyaping@ymail.com (Yaping Mao), echeng@oakland.edu (Eddie Cheng), wangzhao@mail.bnu.edu.cn (Zhao Wang), guozhiweic@yahoo.com (Zhiwei Guo)